Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
TOTAL	

Level 2 Certificate in Further Mathematics
June 2012

Further Mathematics 8360/2
Level 2
Paper 2 Calculator

Friday 1 June 2012 1.30 pm to 3.30 pm

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

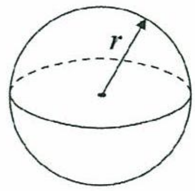
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

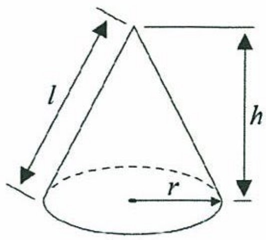
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



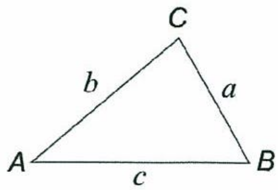
In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\qquad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

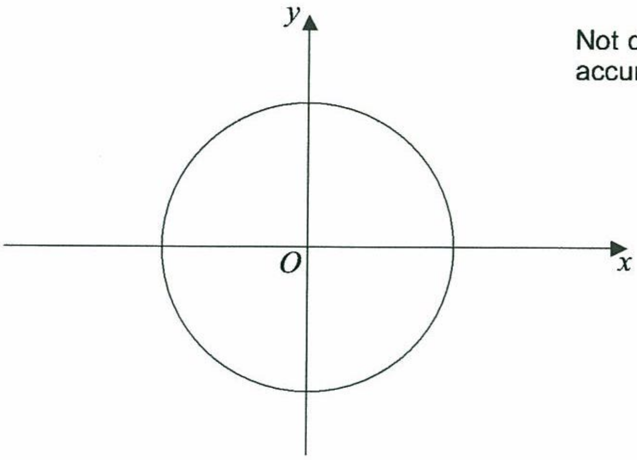
Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Here is a sketch of the circle $x^2 + y^2 = 36$



Not drawn accurately

Work out the circumference of the circle.

$x^2 + y^2 = 6^2$ is the equation of a circle centred at $(0, 0)$ with radius 6. $\therefore$ Circumference of circle $= 2\pi r = 2\pi(6) = 12\pi = 37.7$ (3 s.f.).

Answer 12π or 37.7 (3 s.f.). *(3 marks)*

Turn over for the next question

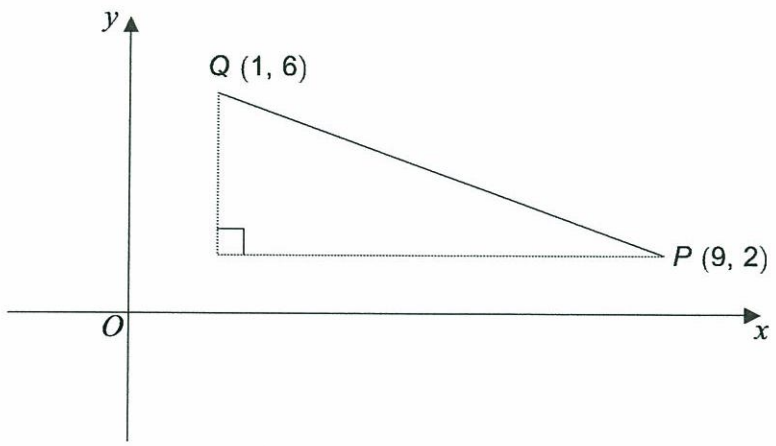
3

2 $y = 5x^3 - 4x^2$

Work out $\frac{dy}{dx}$.

$\frac{dy}{dx} =$ $15x^2 - 8x$ *(2 marks)*

3



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Work out the length of *PQ*.
Give your answer to 3 significant figures.

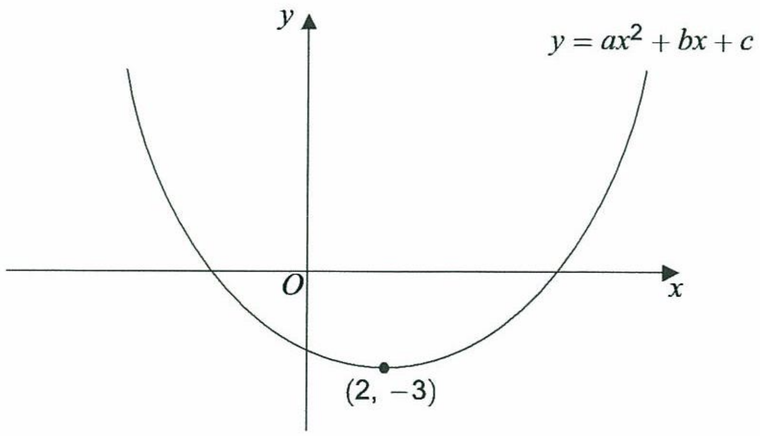
$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(9-1)^2 + (2-6)^2}$

$= \sqrt{8^2 + (-4)^2} = \sqrt{80} = 8.94$ (3s.f.).

PQ = 8.94 (3s.f.). *(4 marks)*

Do not write outside the box

4 A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is (2, -3).



For the sketch shown, circle the correct answer in each of the following.

4 (a) The value of a is

zero positive negative *(1 mark)*

4 (b) The value of c is

zero positive negative *(1 mark)*

4 (c) The solutions of $ax^2 + bx + c = 0$ are

both zero both positive both negative one positive and one negative *(1 mark)*

4 (d) The **number** of solutions of $ax^2 + bx + c = -6$ is

0 1 2 3 *(1 mark)*

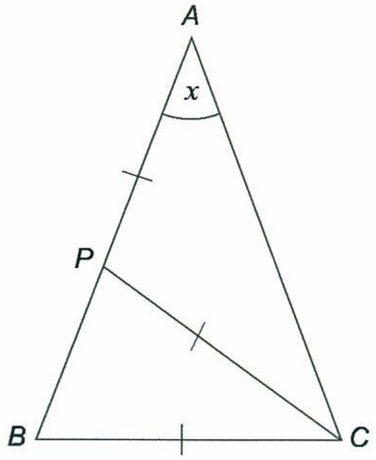
4 (e) The equation of the tangent to $y = ax^2 + bx + c$ at (2, -3) is

$x = 2$ $y = 2$ $x = -3$ $y = -3$ *(1 mark)*

11

Turn over ▶

5 ABC is a triangle.
P is a point on AB such that $AP = PC = BC$
Angle $BAC = x$



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5 (a) Prove that angle $ABC = 2x$

$ACP = x$ and $BPC = ABC$ (Base angles of an isosceles triangle are equal). $APC = 180 - 2x$ and $BPC = 180 - (180 - 2x) = 2x = ABC$

Other theorems used: Angles of a triangle and angles across a straight line add to $180°$.

(3 marks)

5 (b) You are also given that $AB = AC$

Work out the value of x.

$A\hat{B}C = A\hat{C}B \Rightarrow 2x = 180 - 4x + x$

$\Rightarrow 2x = 180 - 3x$

$\Rightarrow 5x = 180 \quad \therefore x = \frac{180}{5} = 36°$

$x =$ 36° degrees *(3 marks)*

Do not write outside the box

6 (a) Expand $3x(2x - 5y)$

Answer $6x^2 - 15xy$ *(2 marks)*

6 (b) Expand and simplify $(3x + 2y)(3x - 4y)$

$9x^2 - 12xy + 6xy - 8y^2$

$= 9x^2 - 6xy - 8y^2$

Answer $9x^2 - 6xy - 8y^2$ *(3 marks)*

6 (c) Work out the ratio $(3x + 2y)(3x - 4y) : 3x(2x - 5y)$ when $y = 0$

Give your answer as simply as possible.

$(3x + 2(0))(3x - 4(0)) : 3x(2x - 5(0))$

$= 3x(3x) : 3x(2x)$

$= 3x : 2x = 3 : 2$

Answer 3 : 2 *(2 marks)*

7 $1 \leqslant m \leqslant 5$ and $-9 \leqslant n \leqslant 2$

7 (a) Work out an inequality for $m + n$.

$1 + (-9) \leqslant m + n \leqslant 5 + 2$

$\Rightarrow -8 \leqslant m + n \leqslant 7$

Answer $-8 \leqslant m + n \leqslant 7$ *(2 marks)*

7 (b) Work out an inequality for $(m + n)^2$.

$(m+n)^2 \geqslant 0$ and $|m+n| \leqslant 8 \Rightarrow (m+n)^2 \leqslant 64$

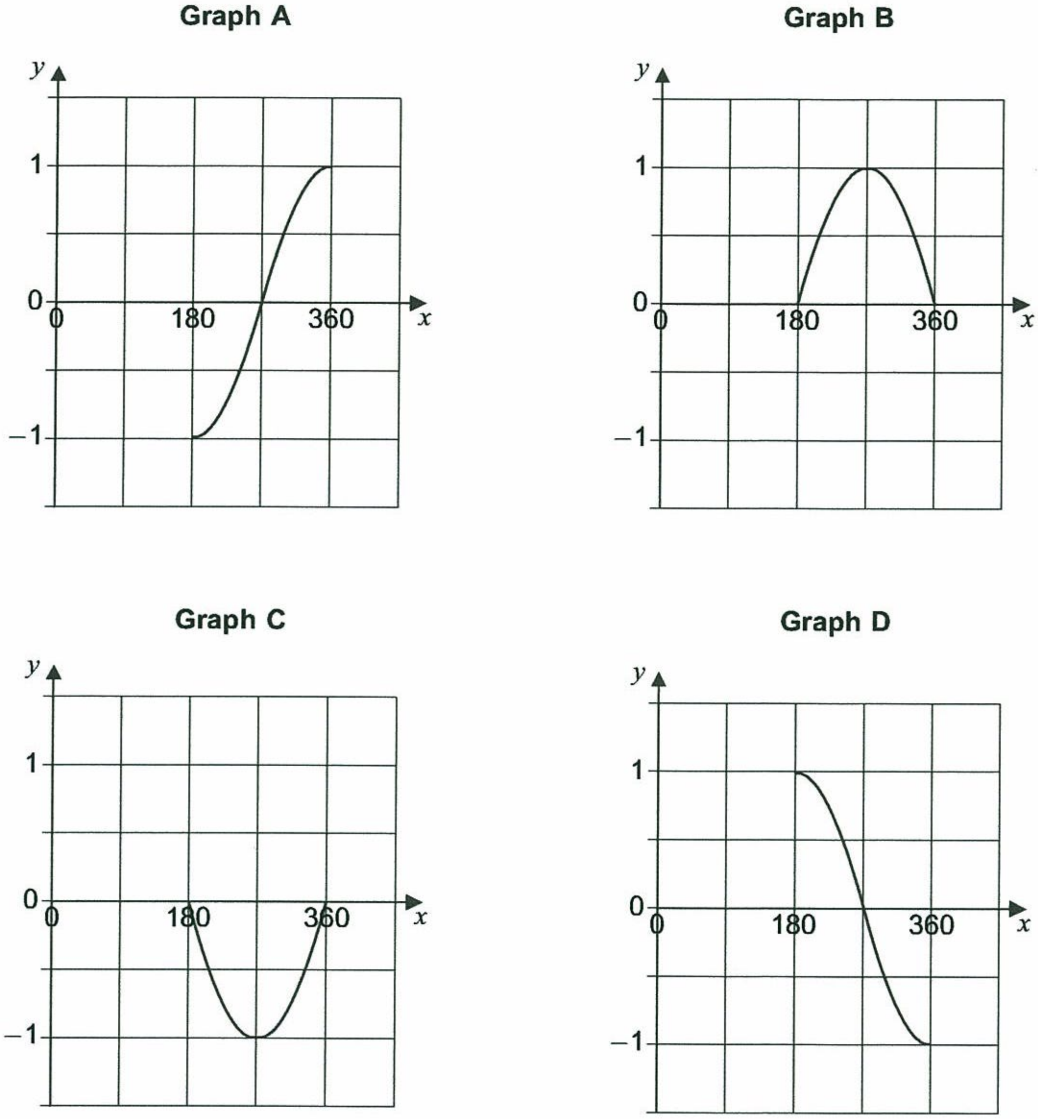
$\therefore 0 \leqslant (m+n)^2 \leqslant 64$

Answer $0 \leqslant (m + n)^2 \leqslant 64$ *(2 marks)*

17

Turn over ▶

8 Four graphs are shown for $180° \leqslant x \leqslant 360°$

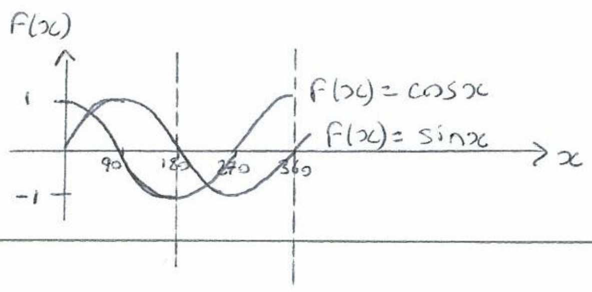


8 (a) Which graph is $y = \sin x$?

Graph ....C........ *(1 mark)*

8 (b) Which graph is $y = \cos x$?

Graph ....A........ *(1 mark)*



Do not write outside the box

9 Here is a formula.

$$5t + 3 = 4w(t + 2)$$

9 (a) Rearrange the formula to make t the subject.

$5t + 3 = 4wt + 8w$

$5t - 4wt = 8w - 3$

$t(5 - 4w) = 8w - 3$

$\therefore t = \dfrac{8w - 3}{5 - 4w}$

Answer $t = \dfrac{8w-3}{5-4w}$ (4 marks)

9 (b) Work out the exact value of t when $w = -\frac{1}{8}$

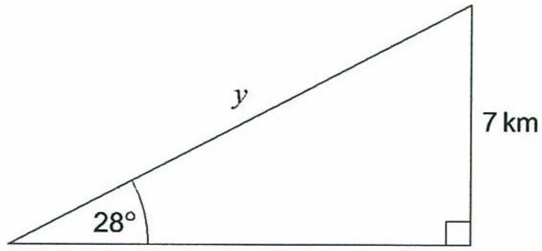
Give your answer in its simplest form.

$t = \dfrac{8\left(-\frac{1}{8}\right) - 3}{5 - 4\left(-\frac{1}{8}\right)} = \dfrac{-1-3}{5+\frac{1}{2}} = \dfrac{-4}{11/2} = -\dfrac{8}{11}$

$t =$ $-\dfrac{8}{11}$ (3 marks)

9

10 An aircraft flies y kilometres in a straight line at an angle of elevation of 28°.
The gain in height is 7 kilometres.



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Work out the value of y.

$\sin 28° = \frac{7}{y}$

$\Rightarrow y = \frac{7}{\sin 28°} = 14.9$ km (3 s.f.).

$y =$ 14.9 km **(3 marks)**
(3 s.f.).

11 A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\frac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

Volume of sphere, $V_S = \frac{4}{3}\pi x^3$

and $V_{Hemisphere} = \frac{1}{2}\left(\frac{4}{3}\right)\pi y^3 = \frac{2}{3}\pi y^3$

Since $V_S = V_H$, $\frac{4}{3}\pi x^3 = \frac{2}{3}\pi y^3 \Rightarrow \frac{y^3}{x^3} = \frac{\frac{4}{3}\cancel{\pi}}{\frac{2}{3}\cancel{\pi}}$

$\Rightarrow \left(\frac{y}{x}\right)^3 = 2 \Rightarrow \frac{y}{x} = \sqrt[3]{2}$ or $2^{\frac{1}{3}}$

$\frac{y}{x} =$ $2^{\frac{1}{3}}$ **(3 marks)**

Do not write outside the box

12 Expand and simplify $(t+4)^3$

$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$ where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$(t+4)^3 = (t+4)(t+4)(t+4) = (t^2+8t+16)(t+4)$

$= t^3 + 12t^2 + 48t + 64$. Alternatively, using the summation formula above and/or Pascal's triangle to assist, $(t+4)^3 = t^3 + 3t^2(4) + 3t(4^2) + 4^3$

$= t^3 + 12t^2 + 48t + 64$

Pascal's triangle

1

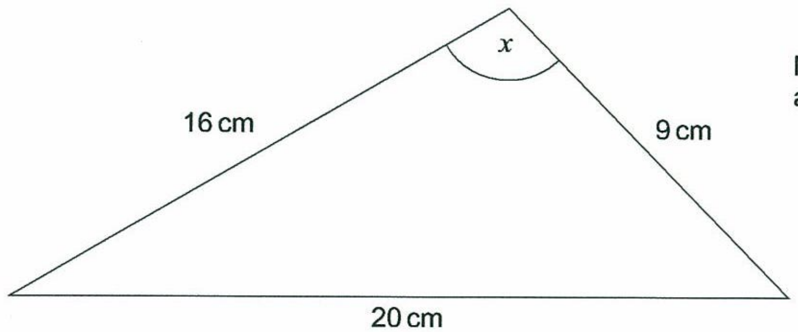
1 1

1 2 1

→ 1 3 3 1

Answer $t^3 + 12t^2 + 48t + 64$ *(3 marks)*

13



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Work out angle x. Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$

$x = \cos^{-1}\left(\frac{16^2 + 9^2 - 20^2}{2(16)(9)}\right)$

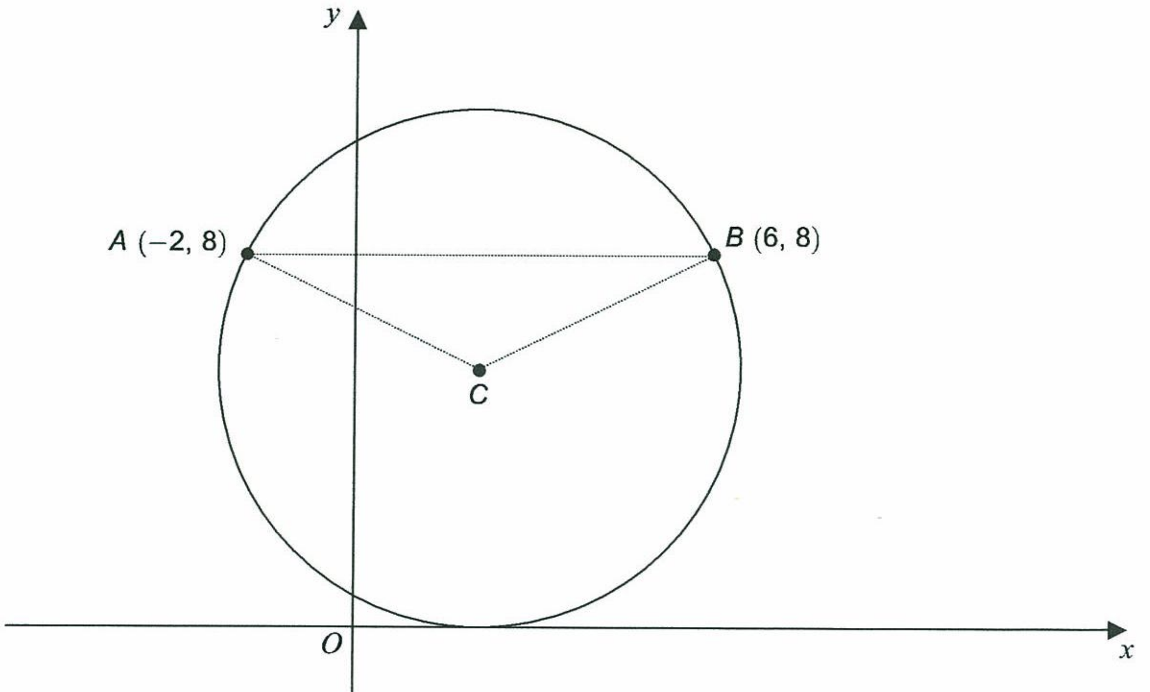
$= 102.6°$ (1 d.p.).

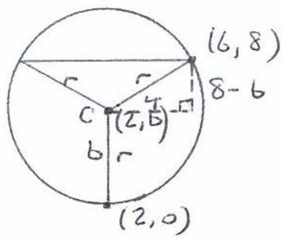
$x =$ 102.6 degrees (1 d.p.). *(3 marks)*

12

Turn over ▶

14 The sketch shows a circle, centre C, radius 5.
The circle passes through the points A $(-2, 8)$ and B $(6, 8)$.
The x-axis is a tangent to the circle.





Work out the equation of the circle.

Let $C = (a, b)$. $a = \frac{6 + (-2)}{2} = 2$.

Then $b^2 = (8 - b)^2 + 4^2$ (Pythagoras)

$\Rightarrow b^2 = 64 - 16b + b^2 + 16$

$\Rightarrow 16b = 80 \Rightarrow b = \frac{80}{16} = 5 \therefore (a, b) = (2, 5)$

and equation of circle is given by $(x - 2)^2 + (y - 5)^2 = 5^2$

Answer $(x - 2)^2 + (y - 5)^2 = 25$ *(4 marks)*

Do not write outside the box

15 (a) $f(x) = 3x - 5$ for all values of x.

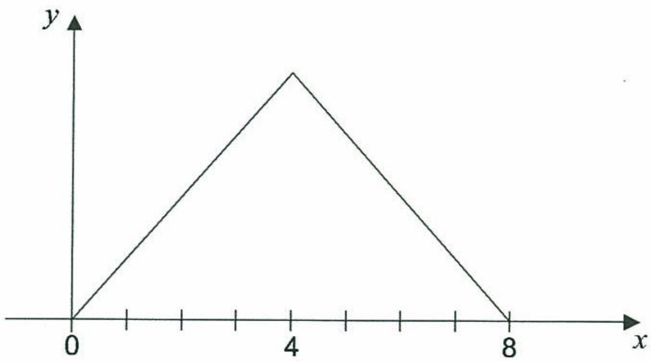
Solve $f(x^2) = 43$

$3x^2 - 5 = 43$

$\Rightarrow x = \pm\sqrt{\frac{43+5}{3}} = \pm\sqrt{16} = \pm 4$

Answer $x = 4$ or -4 *(4 marks)*

15 (b) A sketch of $y = g(x)$ for domain $0 \leqslant x \leqslant 8$ is shown.



The graph is symmetrical about $x = 4$
The range of $g(x)$ is $0 \leqslant g(x) \leqslant 12$

Work out the function $g(x)$.

For $0 \leq x \leq 4$ $g(x) = \left(\frac{12-0}{4-0}\right)x = 3x$

For $4 < x \leq 8$, $g(x) = \left(\frac{0-12}{8-4}\right)x + c$

$= -3x + c$ passing through $(8, 0)$

$\Rightarrow 0 = -3(8) + c \Rightarrow c = 24$ $\therefore g(x) = -3x + 24$

$g(x) =$ $3x$ $0 \leqslant x \leqslant 4$

$-3x + 24$ $4 < x \leqslant 8$

(5 marks)

13

Turn over ▶

16 (a) Use the factor theorem to show that $(x - 1)$ and $(x - 4)$ are factors of $x^3 - 21x + 20$

Factor theorem: $(x-a)$ is a factor of $f(x) \Longleftrightarrow f(a) = 0$. So for $f(x) \equiv x^3 - 21x + 20$, $(x-1)$ and $(x-4)$ are factors if $f(1) = 0$ and $f(4) = 0$

$f(1) = 1^3 - 21(1) + 20 = 0$ & $f(4) = 4^3 - 21(4) + 20 = 64 - 84 + 20 = 0$

(2 marks)

16 (b) Show that $(x - 1)$ and $(x - 4)$ are also factors of $x^3 - 10x^2 + 29x - 20$

$f(1) = 1^3 - 10(1)^2 + 29(1) - 20 = 0$ and

$f(4) = 4^3 - 10(4)^2 + 29(4) - 20 = 64 - 160 + 116 - 20 = 0$. $\therefore$ $(x-1)$ and $(x-4)$ are factors of $x^3 - 10x^2 + 29x - 20$

(2 marks)

16 (c) Hence, simplify fully $\dfrac{x^3 - 21x + 20}{x^3 - 10x^2 + 29x - 20}$

Use polynomial division or 'inductive third factor term inspection' to show that

$x^3 - 21x + 20 \equiv (x-1)(x-4)(x+5)$

and $x^3 - 10x^2 + 29x - 20 \equiv (x-1)(x-4)(x-5)$

$\therefore \dfrac{x^3 - 21x + 20}{x^3 - 10x^2 + 29x - 20} \equiv \dfrac{\cancel{(x-1)}\cancel{(x-4)}(x+5)}{\cancel{(x-1)}\cancel{(x-4)}(x-5)}$

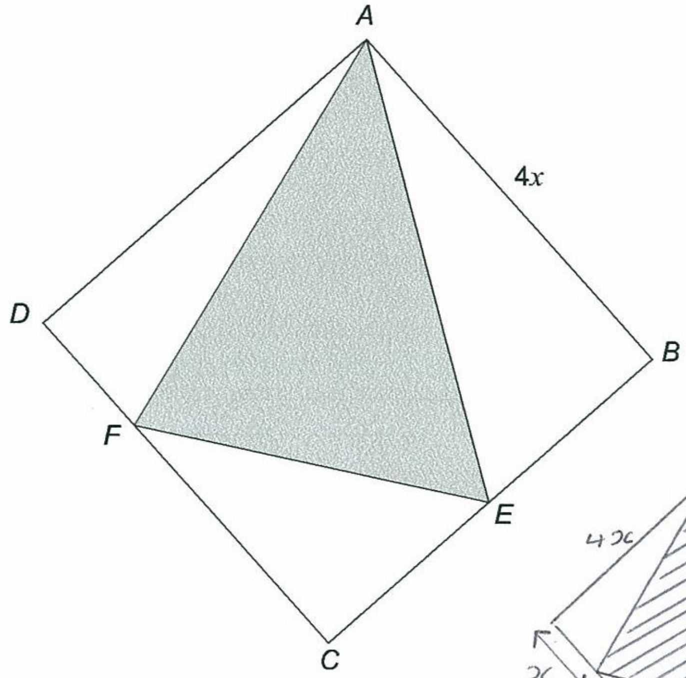
$\equiv \dfrac{x+5}{x-5}$

Answer $\dfrac{x+5}{x-5}$ *(3 marks)*

Do not write outside the box

17 *ABCD* is a square of side length $4x$.

E is the midpoint of *BC*.
$DF : FC = 1 : 3$



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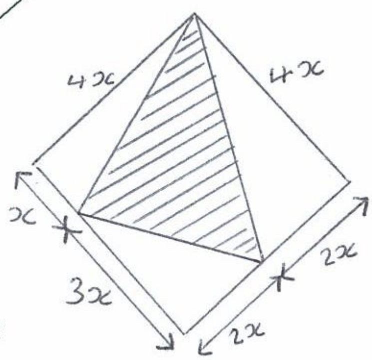


You are given that

area of triangle $AEF = kx^2$

Work out the value of k.

Area of triangle AEF $= 4x(4x) - \frac{1}{2}(2x)(4x)$
$- \frac{1}{2}(2x)(3x) - \frac{1}{2}x(4x) = 16x^2 - 4x^2 - 3x^2$
$- 2x^2 = 7x^2$
$\therefore k = 7$

$k =$ 7 *(5 marks)*

12

Do not write outside the box

18 $(x-5)^2 + a \equiv x^2 + bx + 28$

Work out the values of a and b.

$(x-5)^2 + a = x^2 - 10x + 25 + a \equiv x^2 + bx + 28$

$\Rightarrow b = -10$ and $25 + a = 28$, i.e. $a = 3$

$a =$ 3 $b =$ −10 *(3 marks)*

19 Solve the simultaneous equations

$$x + y = 4 \quad \cdots\cdots \text{①}$$
$$y^2 = 4x + 5 \quad \cdots\cdots \text{②}$$

Do **not** use trial and improvement.

From ①, $x = 4 - y$. In ②, $y^2 = 4(4-y) + 5$

$\Rightarrow y^2 = 16 - 4y + 5$

$\Rightarrow y^2 + 4y - 21 = 0$

$\Rightarrow (y+7)(y-3) = 0$

$\Rightarrow y = -7$ or $y = 3$

In ①, $x = 4 - (-7) = 11$ or $x = 4 - 3 = 1$

$\therefore (x, y) = (11, -7)$ or $(1, 3)$

Answer $(x, y) = (11, -7)$ or $(1, 3)$ *(6 marks)*

20 For what values of x is $y = 150x - 2x^3$ an increasing function?

$y = f(x)$ is an increasing function for all values of x which satisfy $\frac{dy}{dx} > 0$

For $y = 150x - 2x^3$, $\frac{dy}{dx} = 150 - 6x^2$

$150 - 6x^2 > 0 \Rightarrow 6x^2 < 150$

$\Rightarrow x^2 < 25 \Rightarrow |x| < 5$

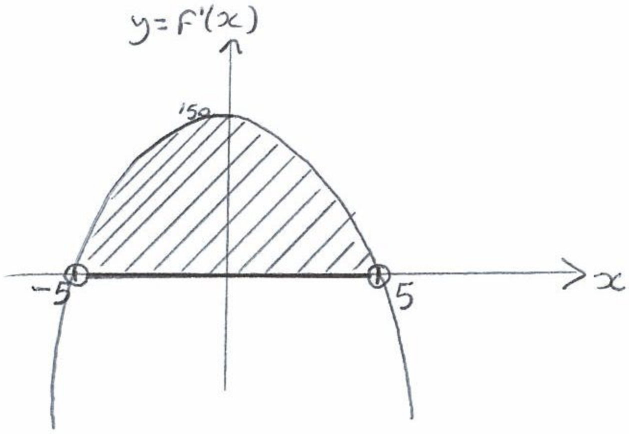
$\therefore x > -5$ and $x < 5$, i.e. $-5 < x < 5$

Answer $-5 < x < 5$ *(4 marks)*

Turn over for the next question

To visualise this, consider the sketch of

$f'(x) = -6x^2 + 150$



Now we can see clearly from the sketch that $f'(x) > 0$ – and thus, $f(x)$ an increasing function – in the bounded interval for x given by

$-5 < x < 5$.

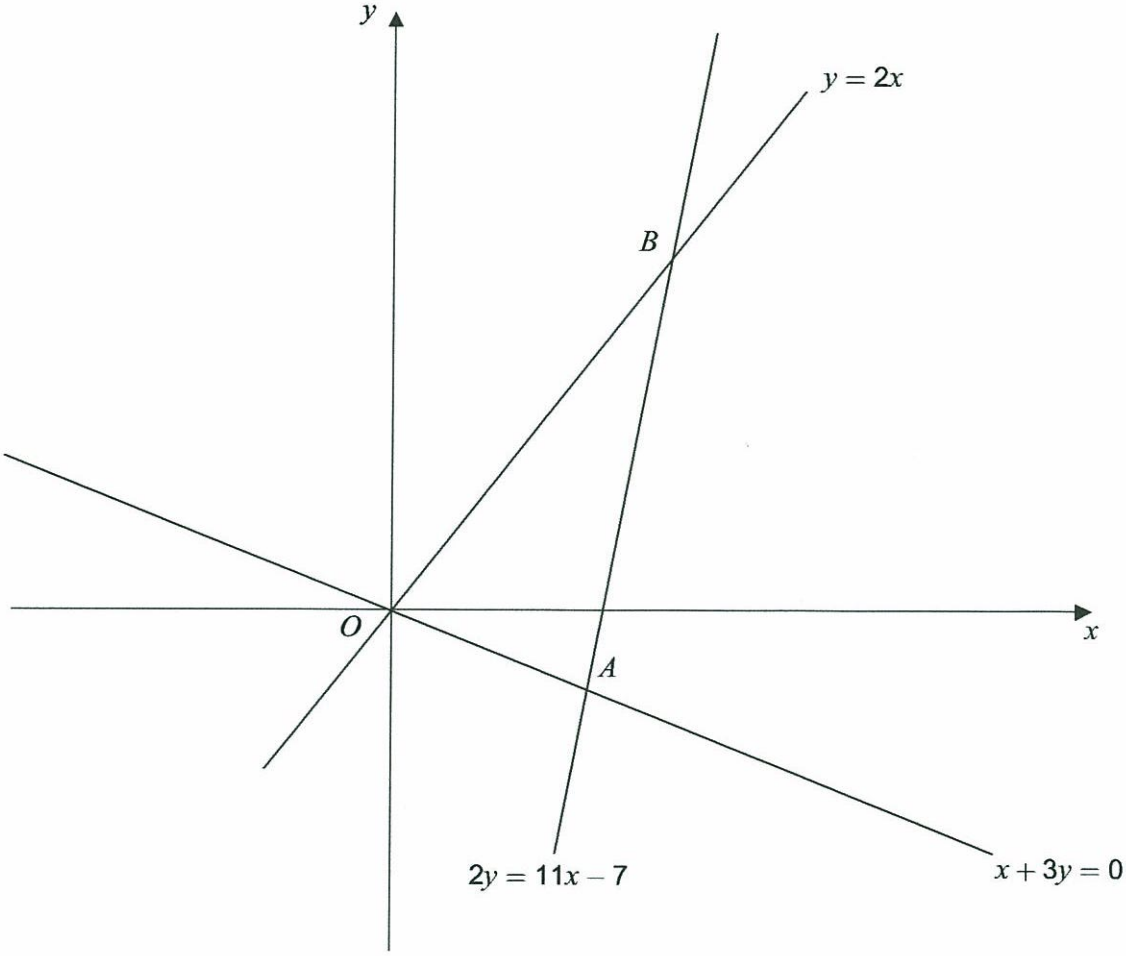
13

Do not write outside the box

21 The equations of three straight lines are

$y = 2x$ $\qquad x + 3y = 0$ $\qquad 2y = 11x - 7$

The lines intersect at the points O, A and B as shown on this sketch.



Show that length OB = length AB

$y = 2x \ldots\ldots ①$ $\quad 2y = 11x - 7 \ldots\ldots ②$ $\quad x + 3y = 0 \ldots\ldots ③$

For intersection point A, consider ② and ③.

From ③, $x = -3y$. In ②, $2y = 11(-3y) - 7$

$\Rightarrow 2y = -33y - 7 \Rightarrow y = \frac{-7}{35} = -\frac{1}{5} \propto x = -3\left(-\frac{1}{5}\right) = \frac{3}{5}$

$\therefore A = \left(\frac{3}{5}, -\frac{1}{5}\right)$

For intersection point B, consider ① and ②.

Substituting ① into ② yields $2(2x) = 11x - 7$

$\Rightarrow 4x = 11x - 7 \Rightarrow 7x = 7 \Rightarrow x = 1$

In ①, $y = 2(1) = 2$. $\therefore B = (1, 2)$

Length OB is given by $\sqrt{1^2 + 2^2} = \sqrt{5}$

and length AB is given by $\sqrt{\left(2 - \left(-\frac{1}{5}\right)\right)^2 + \left(1 - \frac{3}{5}\right)^2}$

$= \sqrt{\left(\frac{11}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \sqrt{\frac{121}{25} + \frac{4}{25}} = \sqrt{\frac{125}{25}} = \sqrt{5}$.

$\therefore$ Length OB = length AB.

(6 marks)

Turn over for the next question

6

22 The transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps point P to point Q.

The transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ maps point Q to point R.

Point R is $(-4, 3)$.

Work out the coordinates of point P.

Let $P = \begin{pmatrix} x \\ y \end{pmatrix}$. Then $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}$

So $Q = \begin{pmatrix} -y \\ -x \end{pmatrix}$. Then $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -y \\ -x \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$

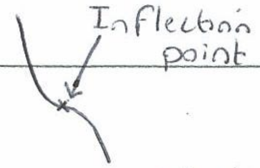
So $R = \begin{pmatrix} -y \\ x \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$\Rightarrow x = 3$ and $y = 4$.

$\therefore P = (x, y) = (3, 4)$

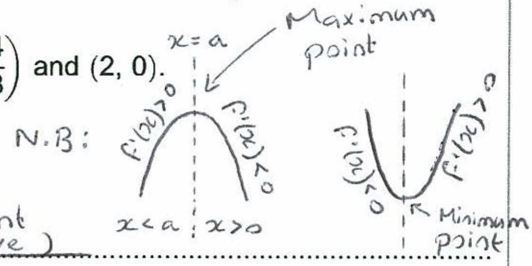
Answer (3 , 4) **(5 marks)**

Do not write outside the box



23 The curve $y = f(x)$ is such that $\dfrac{dy}{dx} = -x(x-2)^2$

The stationary points of the curve are at $\left(0, \dfrac{4}{3}\right)$ and $(2, 0)$.



Determine the nature of each stationary point.
You **must** show your working.

x	$f'(x)$	Sign of gradient (+ve or −ve)
−0.1	0.441	+
0.1	−0.361	−
1.9	−0.019	−
2.1	−0.021	−

$\therefore \left(0, \frac{4}{3}\right)$ is a maximum stationary point.

$\therefore (2,0)$ is an inflection point, i.e. a point where the curve changes from concave to convex (in this case) or vice versa.

(4 marks)

END OF QUESTIONS

N.B: If the second derivative, $f''(x) = 0$, then you have an inflection point. However, not all inflection points are stationary, so be careful here. If $f''(a) = 0$, then $x = a$ is an inflection point but not necessarily stationary. To check whether $x = a$ is stationary simply check whether $f'(a) = 0$. In the question above both $x = 0$ and $x = 2$ are stationary as $f'(0) = 0$ and $f'(2) = 0$.

9

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